DYNAMICS OF SHOCK WAVES IN A LIQUID CONTAINING GAS BUBBLES

S. S. Kutateladze, V. E. Nakoryakov,
V. V. Sobolev, and I. R. Shreiber

UDC 532.593:532.529.5

Results are presented of a numerical solution of the Korteweg–de Vries–Burgers equation that describes the propagation and establishment process for a stationary structure to a shock wave in a gas–liquid medium. Data are obtained on the time for the establishment of a stationary structure of a shock wave, propagation velocity, and amplitude oscillations in the front of the shock wave. Experiments are discussed on the basis of the results obtained for the study of shock waves in a liquid containing gas bubbles.

1. Compression waves of finite amplitude in gas–liquid media have been experimentally and theoretically investigated [1-12].

It has been shown [5, 8] that it is possible for a weak shock wave possessing oscillatory structure to form in a liquid with gas bubbles, given specified relationships between the effective mixture viscosity, intensity of the disturbance, and bubble radius. The case of strong shock waves in such a medium has also been considered [10].

The oscillatory structure of a standing shock wave was calculated in [6] and [8] on the basis of equations for a homogeneous single-velocity model assuming an adiabatic process within the suspension bubbles, and results have been presented [11] of a calculation of a standing wave based on a two-velocity model of the medium assuming a nonpolytropic process.

It has been demonstrated [4, 5] that the evolution of longwave disturbances in a liquid containing gas bubbles can be examined on the basis of the Korteweg–de Vries–Burgers equation, which is a model equation for describing the propagation process for waves of finite amplitude in a medium with weak dispersion and dissipation [13],

$$u_t + uu_x - \eta\, u_{xx} + \beta\, u_{xxx} = 0 \tag{1.1}$$

Here t is time, x is a coordinate, u is the velocity disturbance of the medium, η is the coefficient of effective viscosity of the medium, $\beta = R_0^2 c_0/6\ \alpha_0(1-\alpha_0)$, $(\beta\ c_0^2)^{-1/2}$ is dispersion length, R_0 is the radius of a stable bubble, $c_0 = P^{1/2}\,[\rho_1\alpha_0(1-\alpha_0)]^{-1/2}$ is a low-frequency approximation of the speed of sound in the gas–liquid medium, ρ_1 is liquid density, and α_0 is the initial gas content of the mixture by volume.

Equation (1.1) was written in a frame of reference moving at velocity c_0. When dissipation is due solely to viscous losses at the bubble–liquid boundary, the coefficient of effective viscosity of the mixture has the form

$$\eta = 2\,\nu\,/\,3\,\alpha_0 \tag{1.2}$$

where ν is the kinematic viscosity coefficient of the liquid. A remark regarding the coefficient in (1.2) can be found in [9]; that is, the actual dissipation coefficient in the mixture exceeds by a factor of 20 the coefficient calculated using Eq. (1.2). The coefficient η can be calculated more precisely preceding on the basis of results of [14]:

$$\eta = \delta\omega\, R_0^2\,/\,6\,\alpha_0\ \ 1 - \alpha_0)$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 67–71, September-October, 1974. Original article submitted April 15, 1974.

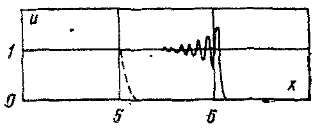

Fig. 1

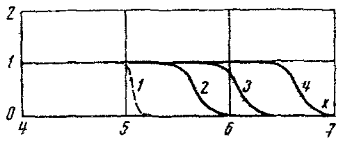

Fig. 2

where δ is the damping decrement, which may be represented in the form of a sum of decrements due to thermal dissipation, acoustic radiation losses, and losses calculated in [14]; ω is the ripple frequency of a bubble, which for weak shock waves is close to the resonance frequency of the bubble calculated using the equilibrium values of pressure and radius.

The contribution to dissipation of losses caused by thermal effects and radiation may be substantial.

The stationary solutions of Eq. (1.1) of the form $u = u(x - Vt)$, which describes the structure of the front, are obtained by integrating the ordinary differential equation

$$\beta u''' - \eta u'' + u'(u - V) = 0 \tag{1.3}$$

where $V = \Delta u/2$, Δu being the velocity discontinuity in the shock wave.

The existence criterion for shock waves with oscillatory structure, following Eq. (1.3), has the form [13]

$$\eta (2 \beta \Delta u)^{-1/2} < 1 \tag{1.4}$$

The criterion (1.4) cannot hold in the course of the establishment of a standing front. The penetration probability for a shock wave with oscillatory structure into a gas–liquid mixture has been confirmed experimentally [4, 6-8, 10, 12].

It has been proposed [11] that times somewhat greater than were realized in these experiments are required for a shock wave to attain a standing structure.

In this work we shall investigate the process by which a shock wave is established and compare it to available experimental results.

2. The process by which a shock wave is established was studied on the basis of the decay problem for an arbitrary discontinuity by means of a numerical integration of Eq. (1.1).

The initial condition for Eq. (1.1) was selected in the form of a step function of the form

$$\begin{aligned} u(x, 0) &= u_0 \text{ m/sec}, \ x \leqslant x_0 \\ u(x, 0) &= u_0 \exp[-(x - x_0^2) / e^2], \ x > x_0 \end{aligned} \tag{2.1}$$

The slope of the discontinuity front can be regulated by varying the parameter. The coordinates and form of the step function are connected by the broken line in Fig. 1. Moreover, numerical experiments with finite initial distributions of the form

$$\begin{aligned} u(x, 0) &= u_0 \exp[-(x - x_1)^2 / e_1^2], \ 0 \leqslant x \leqslant x_1 \\ u(x, 0) &= u_0, \ x_1 < x \leqslant x_0 \\ u(x, 0) &= u_0 \exp[-(x - x_0)^2 / e_2^2], \ x > x_0 \end{aligned} \tag{2.2}$$

were carried out.

The initial disturbance (2.1) approximately corresponds to the conditions of previous experiments [8] and the distribution of the form (2.2), to other experiments [4, 6, 7], where the high-pressure chamber had limited volume.

The dispersion and dissipation coefficients were $\beta = 10^{-4}$ m^3/sec and $\eta = 10^{-3}$ m^2/sec. These values of the coefficients roughly correspond to the conditions of previous experiments [4, 6-8] ($u_0 = 1$ m/sec, $l = 0.08$ m, $l_1 = 0.04$ m, and $l_2 = 0.4$ m).

The results of the calculation are presented in dimensional form to facilitate their comparison with the experimental results.

Equation (1.1) was approximated by an explicit three-layer finite-difference scheme of second order with respect to coordinate and time. The integration step was as follows: for coordinate, $h = 0.01$ m and for time, $\tau = 0.8 \cdot 10^{-3}$ sec.

Results are presented in Fig. 1 of a numerical integration of Eq. (1.1) with initial condition (2.1); the disturbance profile is depicted for $t = 2$ sec. A standing shock wave with oscillatory structure was estab-

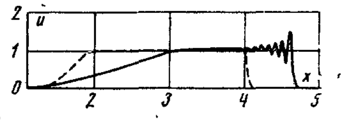

Fig. 3

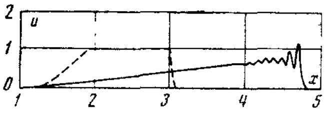

Fig. 4

lished in about 1.2 sec for a height of a step $u_0 = 1$ msec for these values of the parameters. The velocity of the standing shock wave in the frame of reference moving at velocity c_0 was 0.5 u_0, which agrees with the results of an investigation of Eq. (1.3) [13, 15]. The distance $x = (c_0 + 0.5\ u_0)t$ in which a stationary structure of the shock wave is established was measured to tens of meters.

We can agree with [11] that only nonstationary shock waves have been investigated in all previous experiments.

It is evident from the results of a numerical integration of Eq. (1.1) that the oscillations appearing in the front of the shock wave in the course of the evolution of the initial discontinuity have higher amplitudes than at the moment when the stationary profile is established.

The oscillations are smoothed as the shock wave propagates, and at the moment when the stationary structure is established the greatest amplitude of the first oscillation exeeds the initial disturbance and is given by $u_{max} = 1.42\ u_0$.

A numerical experiment with $\beta = 10^{-4}$ m^3/sec and $\eta = 3 \cdot 10^{-2}$ m^2/sec was carried out to verify whether the existence criterion for an oscillatory structure in a shock wave (1.4) holds. Equation (1.4) is evidently not satisfied for such a high value of the viscosity coefficient.

The results of a numerical solution of Eq. (1.1) are depicted in Fig. 2 (curves 1-4 correspond to $t = 0$, 1.2, 2, and 3.2 sec). It is evident that a shock profile of monotonic structure forms. Criterion (1.4) yields a correct representation of the relation of the magnitudes of the parameters that characterize the establishment of an oscillatory structure to a shock wave.

Let us now consider the evolution of an initial distribution of the form (2.2).

The pulse length on a plateau section was taken as $\Delta x = (x_0 - x_1) = 2$m and $x = (x_0 - x_1) = 1$m. The initial stage of evolution of the disturbance (2.2) can be predicted proceeding on the basis of the results of the preceding problem. In the course of the propagation, a stationary oscillatory structure to the shock wave forms at the leading edge, a trailing edge slope also forms, and the leading edge interacts with the compression wave, as a result of which the typical front of a shock wave is formed, in turn forming a "triangle" with the oscillations from a disturbance of finite extent.

The evolution of the initial distribution (2.2) when $\Delta x = 2$m is depicted in Fig. 3 (the broken line corresponds to $t = 0$ and the solid line, to $t = 1.2$ sec). At time $t = 1.2$ sec the leading edge of the front has already been formed but, unlike the case presented in Fig. 2, the form of the wave is not final.

When $\Delta x = 1$ m at time $t = 3.6$ sec (Fig. 4, solid line) a triangular profile, which can be taken as the steady-state structure of the shock wave, is formed (the broken line in Fig. 2 depicts the initial distribution). The shock wave damps, that is, the amplitude of the leading edge decreases, due to the limitedness of energy applied to the initial disturbance and the presence of dissipation in the medium. In this system the total momentum is maintained, so that the "area" of the disturbance remains constant while the shock wave profile spreads. The nature of the damping of the shock wave is depicted in Fig. 4.

Shock waves have been considered [3] in a gas–liquid medium under a nondissipative formulation on the basis of the Korteweg–de Vries equation, which corresponds to Eq. (1.1) when $\eta = 0$.

Figure 5 depicts the result of a numerical solution of Eq. (1.1) when $\eta = 0$ (here $t = 1.6$ sec). Evidently, for sufficiently long periods of time the disturbance is an expanding region filled with oscillations and no stationary structure of finite extent with a definite number of oscillations exists.

Over the course of time a leading soliton forms whose amplitude reaches twice the height of the initial disturbance. Its velocity of propagation in a frame of reference moving at c_0 is $2u_0/3$.

The results of a numerical analysis of the evolution of the step function (2.1) agree with analytical results [16, 17] obtained for $l = 0$.

We should emphasize that such structures of shock waves do not actually represent the process on the basis of the inviscous equation (1.1). To a still higher degree this is a matter of the use of a linear equation (1.1), in which the analysis of the structure of shock waves was practically constructed in [3] and [8]. In this case the velocity of propagation of the disturbance is c_0, and the maximal amplitude of the oscillations in the front is found equal to $u_{max} = 1.2\ u_0$ m/sec when $\beta = 10^{-4}\ m^3/cm^2$ (cf. Fig. 6 in which the solid line corresponds to t = 2 sec, the broken line to t = 0). The value of u_{max} substantially depends on l.

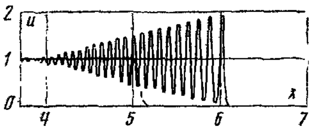

Fig. 5

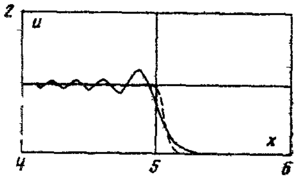

Fig. 6

The investigations conducted showed that the formation of standing shock waves in a gas–liquid medium with a clearly expressed front possessing an oscillatory character or monotonic structure is possible only in the presence of dissipation in the given model. The initial disturbances of the form of the step function (2.1) ensure energy inflow, which compensates the effect of dissipative effects, leading finally to the establishment of a stationary structure of the front of the shock wave. For initial disturbances bounded with respect to coordinate, for example, of type (2.2), the wave damps beginning at some moment of time. A stationary structure of the shock front over some interval of time is possible for a sufficient extent of the initial disturbance in the course of the evolution until the compression wave approaches the leading edge (Figs. 3 and 4). The compression wave then begins to smooth the leading edge and the typical triangular form of the shock wave is formed, which damps over the course of time.

Calculations and experimental investigations into the structure of a shock wave previously conducted [6, 7] correspond to the periods of time during which the compression wave is far from the leading edge, though the observed oscillations in the front of the shock wave are not steady-state due to the small extent of the experimental section. In no experiment in investigating the oscillatory structure of a shock wave [8, 12] have there been observed standing shock waves. Apparently, available experimental data on the structure of shock waves must be compared to results obtained on the basis of solutions of nonstationary equations.

LITERATURE CITED

1. I. J. Cambell and A. S. Pitcher, "Shock waves in a liquid containing gas bubbles," Proc. Roy. Soc., Ser. A, 243, No. 1235 (1958).
2. L. Van Wijngaarden, "On equations of motion for mixtures of fluid and gas bubbles," J. Fluid Mech., 33, No. 3 (1968).
3. L. Van Wijngaarden, "On the structure of shock waves in liquid–bubble mixtures," Appl. Sci. Res., 22, No. 5 (1970).
4. A. P. Burdukov, V. E. Nakoryakov, B. G. Pokusaev, V. V. Sobolev, I. R. Shreiber, "Certain questions in the gasdynamics of a homogeneous model of a two-phase medium," in: Numerical Methods of Continuum Mechanics [in Russian], Vol. 5, No. 2, VTs Sibirsk. Otd., Akad. Nauk SSSR, Novosibirsk (1971).
5. V. E. Nakoryakov, V. V. Sobolev, and I. R. Shreiber, "Longwave disturbances in a gas–liquid mixture," Izv. Akad. Nauk SSSR, MZhG, No. 5 (1972).
6. A. P. Burdukov, V. V. Kuznetsov, S. S. Kutateladze, V. E. Nakoryakov, B. G. Pokusaev, and I. R. Shreiber, "Shock wave in a gas–liquid medium," Prikl. Mekhan. i Tekh. Fiz., No. 3 (1973).
7. S. S. Kutateladze, A. P. Burdukov, V. V. Kuznetsov, V. E. Nakoryakov, B. G. Pokusaev, and I. R. Shreiber, "Structure of a weak shock wave in a gas–liquid medium," Dokl. Akad. Nauk SSSR, 207, No. 2 (1972).
8. L. Noordzij, "Shock waves in bubble–liquid mixtures," Phys. Communs., 3, No. 1 (1971).
9. G. K. Batchelor, "Compression waves in a suspension of gas bubbles in a liquid," Period. Sb. Perev. Inostr. Statei, No. 3 (1968).
10. V. K. Kedrinskii, "Propagation of disturbances in a liquid containing gas bubbles," Prikl. Mekhan. i Tekh. Fiz., No. 4 (1968).
11. R. I. Nigmatulin, N. S. Khabeev, and V. Sh. Shaganov, "Shock waves in a liquid with gas bubbles," Dokl. Akad. Nauk SSSR, 214, No. 4 (1974).
12. B. E. Gel'fand, S. A. Gubin, B. S. Kogarko, and S. M. Kogarko, "Investigation into compression waves in a liquid mixture with gas bubbles," Dokl. Akad. Nauk SSSR, 213, No. 5 (1973).
13. V. I. Karpman, Nonlinear Waves in Dispersing Media [in Russian], Nauka, Moscow (1973).
14. C. Devin, "Survey of thermal radiation and viscous damping of pulsating air bubbles in water," J. Acoust. Soc. Amer., 31, No. 12 (1959).

15. R. Z. Sagdeev, "Collective processes and shock waves in rarefied plasma," in: Problems in Plasma Theory [in Russian], Vol. 4, Atomizdat, Moscow (1964).
16. A. V. Gurevich and L. P. Pitaevskii, "Decomposition of an initial discontinuity in the Korteweg–de Vries equations," Pis'ma Zh. Éksp. i Tekh. Fiz., 17, No. 5 (1973).
17. A. V. Gurevich and L. P. Pitaevskii, "Nonstationary structure of a noncolliding shock wave," Zh. Éksp. i Tekh. Fiz., 65, No. 2 (1973).